

NUMERICAL METHOD FOR CALCULATING THE FILLING OF A MOLD AND HEAT TRANSFER OF A MELT UNDER THE ACTION OF CENTRIFUGAL FORCES

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This paper presents an explicit difference method for solving the conjugate problem of pouring molten metal into a casting mold and its solidification under the action of centrifugal forces with allowance for the free surface.

The development of numerical methods for simulating the processes of filling casting molds with molten metal under the action of centrifugal forces in foundry machines with a vertical rotational axis is an urgent problem for some areas of foundry concerned with the centrifugal casting of shells, bimetal castings, and ingots.

In the present article we propose a rather simple numerical method of calculating the filling of a casting mold with molten metal with account for the free surface and for its subsequent solidification.

In the study of flow and heat transfer, molten metal is usually considered as an incompressible Newtonian fluid [1, 2]. In our case we account for the motion of the molten metal with the free surface using a method based on a modified variant of the SMAC-method [3]. The basic dependent variables are taken to be the velocity vector components u and v , the normalized pressure $\tilde{P} = P/\rho$, the angular velocity of the rotation of fluid ω , and the temperature T . The basic equations describing the flow of molten metal with account for free convection in the Boussinesq approximation, the entrainment of the fluid layers into rotation by the mold, and the heat transfer have the following form in cylindrical coordinates:

$$\frac{\partial u}{\partial \tau} + \frac{1}{r} \frac{\partial}{\partial r} (ru^2) + \frac{\partial}{\partial z} (uv) = - \frac{\partial \tilde{P}}{\partial r} + \nu \left[\frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial}{\partial r} (ru) \right) + \frac{\partial^2 u}{\partial z^2} \right] + \omega^2 r, \quad (1)$$

$$\frac{\partial v}{\partial \tau} + \frac{1}{r} \frac{\partial}{\partial r} (ruv) + \frac{\partial}{\partial z} (v^2) = - \frac{\partial \tilde{P}}{\partial z} + \nu \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial v}{\partial r} \right) + \frac{\partial^2 v}{\partial z^2} \right] - g(1 - \beta(T - T_1)), \quad (2)$$

$$\frac{1}{r} \frac{\partial (ru)}{\partial r} + \frac{\partial v}{\partial z} = 0, \quad (3)$$

$$\frac{\partial \omega}{\partial \tau} + \frac{1}{r} \frac{\partial}{\partial r} (r\omega u) + \frac{\partial}{\partial z} (\omega v) = \nu \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial \omega}{\partial r} \right) + \frac{\partial^2 \omega}{\partial z^2} \right], \quad (4)$$

$$C_{ef} \rho \left(\frac{\partial T}{\partial \tau} + \frac{\partial (uT)}{\partial r} + \frac{\partial (vT)}{\partial z} \right) = \frac{1}{r} \frac{\partial}{\partial r} \left(r\lambda \frac{\partial T}{\partial r} \right) + \frac{\partial}{\partial z} \left(\lambda \frac{\partial T}{\partial z} \right). \quad (5)$$

Release of the heat of phase transition in the solidification of the molten metal within the range of temperatures of the liquidus T_L – solidus T_S is taken into account by the effective heat capacity [4]:

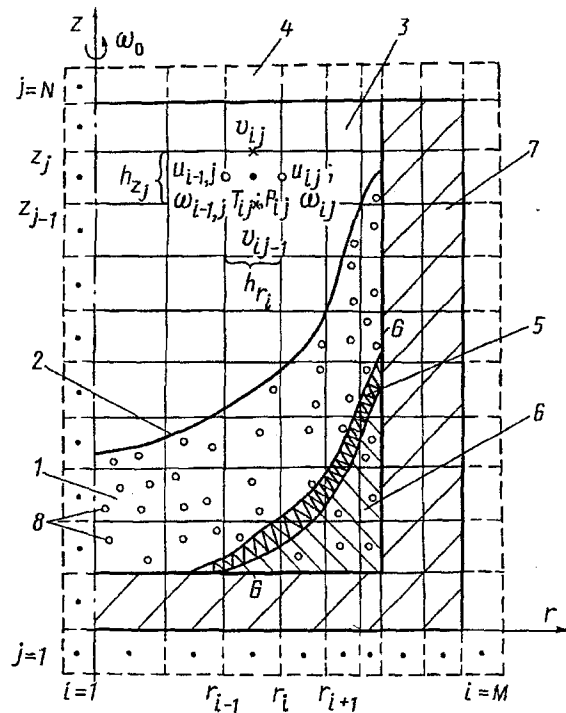


Fig. 1. Scheme of the computational domain: 1) molten zone of the casting; 2) surface cells occupied by molten metal; 3) "vacuum" cells; 4) fictitious cells; 5) two-phase zone of casting; 6) solidified portion of the casting; 7) casting mold; 8) markers.

$$C_{ef} = \frac{\int_{T_S}^{T_L} C(T) dT + L}{T_L - T_S} \quad (6)$$

The numerical implementation of momentum transfer equations (1)-(4) is based on the explicit scheme of splitting by physical factors proposed by O. M. Belotserkovskii [5] with the use of a "staggered" grid. The configuration of the free surface of the fluid is determined by using a Lagrange grid of discrete particles-markers simultaneously with a fixed Euler grid, in which the variables (P , u , v , T) are determined (Fig. 1). According to the method of markers, the particles are distributed not only over the surface, but also throughout the entire volume. The markers move with the velocity calculated by interpolation between the values of the local velocity of the medium in the adjacent cells of the Euler grid. The thermal state equation of molten metal, solidified skin, and casting mold (5) is approximated by Nikitenko's three-layered scaler [6]. On the grid

$$r_{i+1} = r_i + h_{r_i}; \quad z_{j+1} = z_j + h_{z_j}; \quad (7)$$

$$i = 1, 2, \dots, M; \quad j = 1, 2, \dots, N; \quad \tau_{n+1} = \tau_n + \Delta\tau; \quad n = 0, 1, \dots, \infty,$$

it is worthwhile to use cells of equal volumes; this is associated with a discrete representation of a continuous medium determined by the location of the markers. For this purpose, we find the coordinates of the boundaries of the cells from the solution of the system of equations

$$r_i^{k+1} = r_i^k + \gamma \left(\sqrt{\left((r_{i-1}^2 + r_{i+1}^2)/2 \right)} - r_i^k \right), \quad (8)$$

where γ is the iterative parameter; k is the iteration number.

The approximation of the system of equations (1)-(5) can be represented in the form

$$\delta_r \bar{u} + \frac{1}{r} \delta_r (ru^2) + \delta_z (uv) = \delta_r \left(v \frac{1}{r} \delta_r (ru) \right) + \delta_z (v \delta_z u) + \omega^2 r, \quad (9)$$

$$\delta_r \bar{v} + \frac{1}{r} \delta_r (ruv) + \delta_z (v^2) = \frac{1}{r} \delta_r (vr \delta_r v) + \delta_z (v \delta_z v) - g(1 - \beta(T - T_1)), \quad (10)$$

$$\frac{\tilde{D}}{\Delta \tau} = \frac{1}{r} \delta_r (r \delta_r \tilde{P}) + \delta_z (\delta_z \tilde{P}), \quad D^{n+1} = 0,$$

$$\tilde{D} = \frac{1}{r} \delta_r (r \bar{u}) + \delta_z \bar{v}, \quad (11)$$

$$\delta_r u^{n+1} = -\delta_r \tilde{P}; \quad \delta_r v^{n+1} = -\delta_z \tilde{P}, \quad (12)$$

$$\delta_r \omega + \frac{1}{r} \delta_r (r \omega u) + \delta_z (\omega v) = \delta_r \left(v \frac{1}{r} \delta_r (r \omega) \right) + \delta_z (v \delta_z \omega), \quad (13)$$

$$\delta_r \bar{T} = -\delta_r (u^{n+1} T) - \delta_z (v^{n+1} T), \quad (14)$$

$$C_{ef} \rho \left[\delta_r T^{n+1} (1 + \theta) - \theta \delta_r \bar{T} \right] = -\delta_r (u^{n+1} \bar{T}) - \delta_z (v^{n+1} \bar{T}) + \frac{1}{r} \delta_r (r \lambda \delta_r \bar{T}) + \delta_z (\lambda \delta_z \bar{T}). \quad (15)$$

Here

$$\delta_r \varphi = \frac{1}{\Delta \tau} (\varphi_{ij}^{n+1} - \varphi_{ij}^n); \quad \delta_r \bar{\varphi} = \frac{1}{\Delta \tau} (\bar{\varphi}_{ij} - \varphi_{ij}^n);$$

$$\delta_r \bar{\bar{\varphi}} = \frac{1}{\Delta \tau} (\varphi_{ij}^n - \varphi_{ij}^{n-1}); \quad \varphi = u, v, \omega, T; \quad \bar{\varphi} = \bar{T}; \quad \bar{\bar{\varphi}} = \bar{\bar{T}}.$$

The difference analog for the functions occurring at the center of the cells is written in the following form:

$$\frac{1}{r} \delta_r \left(k \frac{1}{r} \delta_r \varphi \right) = \frac{1}{r_{i-1/2}} \frac{1}{h_{ri}} \left[\frac{2r_i k_{i+1/2}}{h_{ri+1} + h_{ri}} (\varphi_{i+1,j} - \varphi_{ij}) - \frac{2r_{i-1} k_{i+1/2}}{h_{ri-1} + h_{ri}} (\varphi_{ij} - \varphi_{i-1,j}) \right], \quad k_{i+1/2} = \frac{(h_{ri} + h_{ri+1}) \lambda_{ij} \lambda_{i+1,j}}{(h_{ri} \lambda_{i+1,j} + h_{ri+1} \lambda_{ij})},$$

$$\varphi = \tilde{P}, \tilde{T}; \quad k = 1, \lambda;$$

$$\delta_r (u\varphi) = \frac{2}{h_{ri+1} + 2h_{ri} + h_{ri-1}} \left[\frac{2u_{ij}^{n+1}}{h_{ri} + h_{ri+1}} (\varphi_{i+1,j} - \varphi_{ij}) + \frac{2u_{i-1,j}^{n+1}}{h_{ri} + h_{ri-1}} (\varphi_{ij} - \varphi_{i-1,j}) \right], \quad \varphi = T^n, \tilde{T};$$

$$\delta_r \tilde{P} = \frac{2}{h_{ri} + h_{ri+1}} (\tilde{P}_{i+1,j} - \tilde{P}_{ij}).$$

The derivatives with respect to the coordinate z are approximated in the same way as with respect to r . The momentum transfer equations (9), (10), and (13), whose variables are located at the cell boundary, have the following finite-difference formulation:

$$\frac{1}{r} \delta_r (ru\varphi) = \frac{1}{r_i} \frac{2}{h_{ri} + h_{ri+1}} \left[r_{i+1/2} (u_{i+1,j}^n + u_{ij}^n) (\varphi_{i+1,j} + \varphi_{ij}) - \right.$$

$$\begin{aligned}
& - r_{i-1/2} (u_{ij}^n + u_{i-1,j}^n) (\varphi_{ij} + \varphi_{i-1,j}) \Big], \\
\delta_z (v\varphi) &= \frac{1}{h_{zj}} \left[\frac{v_{ij}^n h_{ri} + v_{ij+1}^n h_{ri+1}}{h_{ri} + h_{ri+1}} \frac{\varphi_{ij+1} h_{zj+1} + \varphi_{ij} h_{zj}}{h_{zj} + h_{zj+1}} - \right. \\
& \left. - \frac{v_{ij-1}^n h_{ri} + v_{i+1,j-1}^n h_{ri+1}}{h_{ri} + h_{ri+1}} \frac{\varphi_{ij} h_{zj} + \varphi_{ij-1} h_{zj-1}}{h_{zj} + h_{zj-1}} \right], \\
\delta_r \left(\frac{v}{r} \delta_r (r\varphi) \right) &= \frac{2}{h_{ri} + h_{ri+1}} \left[\frac{v}{r_{i+1/2}} \frac{r_{i+1} \varphi_{i+1,j} - r_i \varphi_{ij}}{h_{i+1}} - \right. \\
& \left. - \frac{v}{r_{i-1/2}} \frac{r_i \varphi_{ij} - r_{i-1} \varphi_{i-1,j}}{h_{ri}} \right], \\
\delta_z (v\delta_z \varphi) &= \frac{1}{h_{zj}} \left[(\varphi_{i,j+1} - \varphi_{ij}) \frac{2v}{h_{zj} + h_{zj+1}} - (\varphi_{ij} - \varphi_{ij-1}) \frac{2v}{h_{zj} + h_{zj-1}} \right], \\
\varphi &= u^n, \omega^n.
\end{aligned}$$

The derivatives for the vertical component v are approximated similarly to the component u .

The kinematic viscosity coefficient ν , taking into account the turbulent character of flow, is taken in the form [7]:

$$\nu = \nu_m + \frac{\Delta}{\text{Re}_\Delta} |\mathbf{V}| + l^2 \left| \frac{\partial \mathbf{V}}{\partial \tau} \right|, \quad (16)$$

where Δ is the determining grid spacing; Re_Δ is the grid Reynolds number ($\text{Re}_\Delta \approx 2$; l is the mixing length (a multiple of the value of the grid spacing Δ is usually used); ν_m is the molecular viscosity.

The effective thermal conductivity of the metal in molten state is defined by the expression

$$\lambda = \lambda_{\text{mol}} + C\rho |\mathbf{V}| \Delta. \quad (17)$$

The equation for the pressure field (10) is elliptic; it is solved at each time step by the method of sequential relaxation [8]. The process of obtaining the solution is considered to be completed, when the following condition is satisfied:

$$\sum_i \sum_j \left| -\frac{\tilde{D}}{\Delta\tau} + \frac{1}{r} \delta_r (r\delta_r \tilde{P}) + \delta_z (\delta_z \tilde{P}) \right| < \psi, \quad (18)$$

where ψ is a small positive number.

The necessary conditions for the stability of solution of the difference momentum transfer equations (8)-(9) are determined by means of the technique of differential approximations:

$$\Delta\tau \leq \frac{2\nu}{\max(u^2, v^2)}. \quad (19)$$

Along with this, another restriction on the integration step can be imposed by the conditions of transfer of the markers on the Euler grid:

$$\Delta\tau \leq \min \left(\frac{h_{ri}}{|u_{ij}|}, \frac{h_{zj}}{|v_{ij}|} \right). \quad (20)$$

The conditions of stability found for the difference equation (14) by means of the maximum principle [9] impose the following restrictions:

$$\Delta\tau \leq \min \left(\frac{1 + 2\Theta}{2 \frac{\lambda}{C\rho} (h_{ri}^{-2} + h_{zj}^{-2})} \right), \quad \Theta \geq 0. \quad (21)$$

Test calculations showed that the admissible accuracy of the solution of the linear heat conduction equation was observed within the limits of variation of the parameter $\Theta = 0...5$.

In our calculations of the processes of filling and heat exchange of a solidifying metal in a cylindrical mold the initial temperature distribution is taken to be homogeneous:

$$T(0, r, z) = T_0. \quad (22)$$

Molten metal enters into the mold with the prescribed velocity

$$u(\tau, r_g, z_g) = u_0; \quad v(\tau, r_g, z_g) = v_0; \quad T(\tau, r_g, z_g) = T_0. \quad (23)$$

On the solid wall of the mold, the conjugation boundary of the two-phase and molten zone, and on the symmetry axis, we assume the condition of zero leakage for the normal velocity component V_{\perp} . The tangential velocity satisfies the condition of free slipping or adhesion.

$$\frac{\partial V_{\parallel}}{\partial n} = 0; \quad V_{\parallel} = 0. \quad (24)$$

Since the tangential velocity component is calculated at a distance of $\Delta/2$ along the normal from the surface, we additionally introduce the cells lying beyond the boundary [10] and determine in them the fictitious velocity:

$$V_{\parallel \text{fic}} = (1 - \gamma) V_{\parallel \text{in}}, \quad (25)$$

where γ is a coefficient determining the type of conditions (24) ($\gamma = 0$ is the slipping condition, $\gamma = 2$ is the complete adhesion, $\gamma = 0...2$ is the partial adhesion); $V_{\parallel \text{in}}$ is the velocity inside the computational domain.

The assignment of velocities on the free surface of the fluid, whose cells are marked, is performed according to the recommendations of [11].

To solve the grid equation (11), we assign the pressures $\tilde{P} = 0$ in "vacuum" cells. The conditions at the boundary between the fluid and the body of the mold or on the moving boundary of the solidification front are prescribed by the known velocity component along the normal to the wall. In this case Eqs. (11)-(12) are transformed in the following manner:

$$\delta_r \tilde{P} = 0, \quad \delta_z \tilde{P} = 0, \quad \delta_r u = 0, \quad \delta_r v = 0. \quad (26)$$

The conditions of heat exchange of molten metal on the free surface in the closed volume of the mold are taken in the form

$$\pm \lambda \frac{\partial T}{\partial n} = 0. \quad (27)$$

Heat removal from the external surface of the mold is described by the conditions of radiative-convective heat exchange:

$$-\lambda \frac{\partial T}{\partial n} = \alpha (T_s - T_{\text{med}}) + \varepsilon \sigma_0 (T_s^4 - T_{\text{med}}^4). \quad (28)$$

Taking into consideration the fact that the grid values of temperatures are located at the center of the cells (see Fig. 1), we determine the surface temperature of the body T_{surf} by the Newton method.

The heat exchange in the zone of contact of the casting metal with the mold surface is taken into account in the approximation of the linearity of temperature distribution over the cross section of the thermal coating and of the gap formed:

$$-\lambda \frac{\partial T}{\partial n} \Big|_{G-0} = -\lambda \frac{\partial T}{\partial n} \Big|_{G+0} = K_{\Sigma} (T_{s1} - T_{s2}), \quad (29)$$

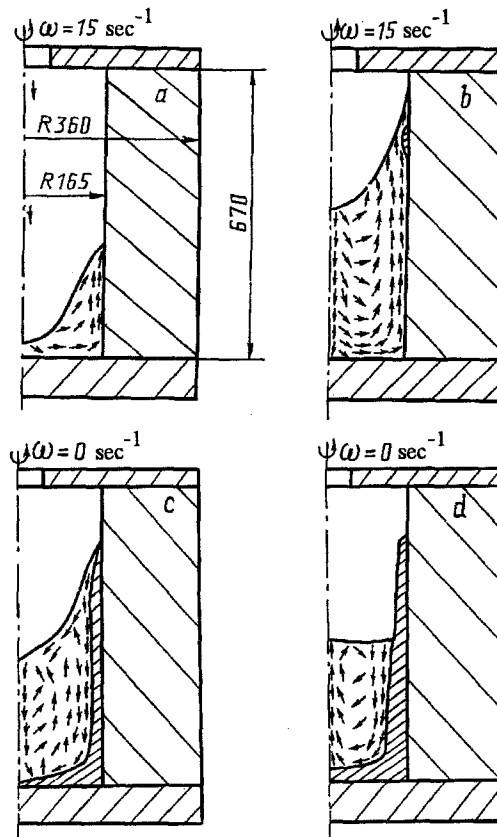


Fig. 2. Dynamics of the filling and solidification of a casting in centrifugal casting: a) $\tau = 10$ sec; b) 60; c) 90; d) 120 sec.

where

$$K_{\Sigma} = \left(\frac{\delta_{\text{coat}}}{\lambda_{\text{coat}}} + \frac{\delta_g}{\delta_g \alpha_{\text{rad}} + \lambda_g} \right)^{-1}, \quad \alpha_{\text{rad}} = \epsilon_{\text{red}} \sigma_0 (T_{s1}^2 + T_{s2}^2) (T_{s1} + T_{s2}).$$

For numerical implementation of the mathematical model we use an additional condition of mass balance of the metal added:

$$\int_0^{\tau} \vartheta_{\text{fil}} d\tau \approx \sum_i \sum_j \pi (r_i^2 - r_{i-1}^2) h_{zj} R_{ij}, \quad (30)$$

where ϑ_{fil} is the volumetric velocity of entering of the metal into the mold;

$$R_{ij} = \begin{cases} 1 - \text{cell labeled by markers,} \\ 0 - \text{«vacuum» cell.} \end{cases}$$

For condition (30) to be satisfied, we prescribed the required quantity of markers, whose number was determined from preliminary calculations; it depends on the grid spacing and the accuracy of the solution of the equation for the pressure field, Eq. (18).

A comparison of the results of test calculations with the well-known solutions for the drop of a liquid column in a basin [3], for the problem of thermal convection in a closed volume [12], and for the capture of a fluid by a rotating cylinder up to the formation of a rotation paraboloid [13] showed sufficient qualitative and quantitative coincidence.

For a comprehensive check of the adequacy of the proposed mathematical model, we conducted a special experiment* in the conditions of the pilot plant of the Scientific-Industrial Association "Chermetmekhanizatsiya." Using specially designed fastenings, a centrifugal casting mold, previously heated to a temperature of 343 K, was attached to the faceplate of a vertical turning lathe used as an industrial model of a vertical-type centrifugal

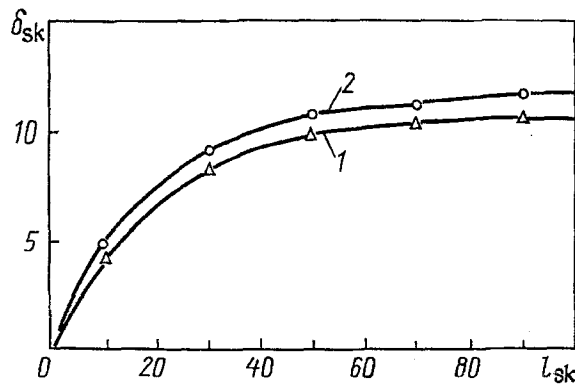


Fig. 3. Dimensions of the solidified skin along the height of casting: 1) experimental data; 2) calculation by the model.

machine. Metal was poured through a filling funnel directly into the casting mold rotated with a frequency of 15 sec^{-1} . The time of filling a 240 kg portion of iron with initial temperature 1593 K was 65 sec. Upon filling, the centrifugal casting mold with the metal, whose free surface took the form of a paraboloid, was rotated for 27 sec. Then in 8 sec the rotation of the casting mold was stopped. Upon complete solidification of the casting, we measured the thickness of the skin in different cross sections over the height of the casting.

For calculations the following characteristics were used: casting mold height 0.67 m; inner and outer radii of the casting mold 0.165 and 0.360 m; the number of markers corresponding to 240 kg of the metal poured was assumed to be equal to 300; the quantity of the grid partitions was 12×21 . The thermophysical parameters of the casting-casting mold system were taken from [14]. An algorithm for the calculation of the process of filling a casting mold and casting solidification is implemented in the Turbo Pascal language. Computations were performed on an IBM PC/AT 386 computer. The time of computing a control variant amounted to 12 h. The integration step was taken to be equal to 0.001 sec.

Figure 2 presents the dynamics of filling a casting mold at different time instants. On pouring, poured, the jet of metal entering from above impacts on the lower end face of the casting mold, turns, and rises up along the lateral surface of the mold (Fig. 2a). As the melt rises under the action of centrifugal forces, the metal cools down appreciably, so that interceptions in the flow nose are formed (Fig. 2b). On stoppage of the mold, the molten portion of the metal begins to descend (Fig. 2d). It should be noted that at this time thermogravitational convection begins to manifest, which was almost completely suppressed by the centrifugal forces during the rotation of the mold.

Comparison of the calculated and experimental data on the thickness of the cast skin (Fig. 3) shows good agreement of the results obtained with the use of a rather coarse grid.

The results of numerical experiments indicate the efficiency of the proposed method for calculating the hydrodynamic problems of filling and heat exchange in casting molds under the action of centrifugal forces and with account of the free surface of the fluid.

NOTATION

r, z , transverse and longitudinal coordinates; u, v , horizontal and vertical velocity components; V , velocity vector; ω , angular velocity of rotation; P , pressure; \bar{P} , normalized pressure; T , temperature; τ , time; g , free fall acceleration; β , coefficient of volumetric expansion; C , heat capacity; λ , thermal conductivity; ρ , density; L , crystallization heat; ν , viscosity; $Re_{\Delta} = |V|h/\nu$, Reynolds grid number; h , grid spacing; l , mixing length in a turbulent flow; Θ , relaxation parameter; α , coefficient of convective heat transfer; γ , coefficient defining the boundary conditions at the solid wall; D , flow divergence; ϑ , volumetric velocity of filling; ε , emissivity; σ_0 , Stefan-Boltzmann constant; δ , thickness of a layer. Indices: r, z, I, j , numbers of grid nodes; n , number of the integration step with

* A. A. Sokol and V. I. Ismagilov took part in the experiments.

respect to time; L , S , temperatures of the liquidus and solidus, respectively; s , temperature on the surface; med , temperature of the medium; 0 , initial state of the system; m , metal; mol , molten state of the metal; red , reduced emissivity in the gap; rad , radiant component of the heat transfer coefficient; g , gas-air gap; $coat$, heat-insulating coating; fil , filling.

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